Normal lines

At every point along a function, the function has a slope that we can calculate. If our function is a straight line, it'll have the same slope at every point. But for any function that isn't a straight line, the slope of the function will change as the value of the function changes.

To find the slope of a function at a particular point, we can take the derivative of the function, and then evaluate it at the point we're interested in. Doing that gives us the slope of the function at the point, but also the slope of the tangent line to the function at that point.

The normal line is the line which is perpendicular to the tangent line at the point where the tangent line intersects the function. Which means that, if the slope of the tangent line is m, then the slope of the normal line is the negative reciprocal of m, or $-1/m$.

In summary, follow the steps below in order to find the equation of the normal line.

1. Take the derivative of the original function, and evaluate it at the given point. This is the slope of the tangent line, which we'll call m.

2. Find the negative reciprocal of m, in other words, find $-1/m$. This is the slope of the normal line, which we'll call n. So $n = -1/m$.

3. Plug n and the given point into the point-slope formula for the equation of the line, $(y - y_1) = n(x - x_1)$.

4. Simplify the equation by solving for y.

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Example

Find the equation of the normal line at the point $(1,9)$.

$$y = 6x^2 + 3$$

Let's follow the steps we just outlined. First, we'll take the derivative of our function, and then evaluate it at $(1,9)$.

$$y'(x) = 12x$$

$$y'(1) = 12(1)$$

$$y'(1) = 12$$

$$m = 12$$

Since $m = 12$, we'll take the negative reciprocal to find n, the slope of the normal line.

$$n = -\frac{1}{12}$$

We'll plug $n = -1/12$ and the point $(1,9)$ into the point-slope formula for the equation of the line,

$$(y - y_1) = m(x - x_1)$$

Once we simplify, we'll have the equation of the normal line to the function at the given point.

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$$y - 9 = -\frac{1}{12}(x - 1)$$

$$12y - 108 = -(x - 1)$$

$$12y - 108 = -x + 1$$

$$12y = -x + 109$$

$$y = \frac{109 - x}{12}$$

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